

# FRW Cosmological Solutions in M-theory

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(MIT-CTP-3050, gr-qc/0011098. November 27, 2000)

## Abstract

We present the canonical and quantum cosmological investigation of a four-dimensional, spatially flat, Friedmann-Robertson-Walker (FRW) model that is derived from the bosonic Neveu-Schwarz/Neveu-Schwarz sector of the low-energy M-theory effective action. We discuss in detail the phase space of the classical theory. We find the quantum solutions of the model and obtain the positive norm Hilbert space of states. Finally, the correspondence between wave functions and classical solutions is outlined.

## 1 Introduction

The search for a theory of quantum gravity constitutes one of the foremost challenges in theoretical high energy physics. The need for quantum gravity finds its roots within Einstein general relativity. Powerful general theorems imply that our universe must have started from an initially singular state with infinite curvature. In such circumstances, where the laws of classical physics break down, it is unclear how any boundary conditions necessary for a description of a dynamical system could have been imposed at the initial singularity. Quantum corrections could then induce a modification of classical general relativity and strongly influence the evolution of the very early universe.

In the last two decades superstring theory[1] has emerged as a successful candidate for the theory of quantum gravity. In cosmology, most of the modifications to general relativity induced by superstring theory are originated by the inclusion of the dilaton, axion and various moduli fields, together with higher curvature terms that are present in the low-energy effective actions. Each of these novel ingredients leads to new cosmological solutions. A remarkable example is given by the so-called pre-big bang scenario[2] that follows from the low-energy effective string action. Different branches of the solution

are related by time reflection and internal transformations –  $O(d, d)$  and, in particular, scale factor duality – that descend from the  $T$ -duality property of the full superstring theory. According to the pre-big bang scenario the universe evolves from a weak-coupled string vacuum state to a radiation-dominated and a subsequent matter-dominated FRW geometry going through a region of strong coupling and large curvature. Although the pre-big bang model has not yet proven able to solve all its pitfalls, such as the existence of a singular boundary that separates the pre- and post-big bang branches[3], it provides a good starting point to investigate high-energy cosmology.

Recently, it has been argued that the five consistent, anomaly free, perturbative formulations of ten-dimensional superstring theories are connected by a web of duality transformations and constitute special points of a large, multi-dimensional moduli space of a fundamental (non-perturbative) theory, called M-theory. Quite interestingly, another point of the moduli space of M-theory corresponds to eleven-dimensional supergravity, which is the low-energy limit of M-theory. Assuming that M-theory is the ultimate theory of quantum gravity, it is natural to explore its cosmological implications. Although our understanding of M-theory is still incomplete, there are hopes that some of the obstacles of dilaton driven inflation in string theory could be overcome within the new theory. The underlying idea is to investigate the dynamics at the extreme weak- and strong-coupling regimes of superstring theory from a M-theory perspective, where the existence of eleven dimensions seems mandatory.

Several approaches to M-theory cosmology have been explored in the literature[4, 5, 7]-[11]. In the framework of the Hořava-Witten model[6], M-theory and cosmology have been combined in the works of Lukas, Ovrut and Waldram[4]. A somewhat related line of research is the brane world by Randall and Sundrum[5], where our four-dimensional universe emerges as the world volume of a three-brane in a higher-dimensional space-time. From a different point of view, Damour and Henneaux have investigated chaotic models[7] and Lu, Maharana, Mukherji and Pope have discussed classical and quantum M-theory models with homogeneous graviton, dilaton and antisymmetric tensor field strengths[8]. Different classes of cosmological solutions that reduce to solutions of string dilaton gravity have been discussed[10]. In particular, a global analysis of four-dimensional cosmologies derived from M-theory and type  $IIA$  superstring theory has been presented by Billyard, Coley, Lidsey and Nilsson (BCLN)[11]. Using the theory of dynamical systems to determine the qualitative behaviour of the solutions, the authors find that fields associated with the Neveu/Schwarz-Neveu/Schwarz (NSNS) and Ramond-Ramond (RR) sectors play a rather crucial role in determining the dynamical behaviour of the solutions. Quite interestingly, for spatially flat FRW models the boundary of the classical physical phase space is a set of invariant submanifolds, where either the axion field is trivial or the RR four-form field strength is dynamically unimportant. This interplay leads to important consequences, as the orbits in the phase space are dominated by the dynamics associated with one, or the other, or both invariant submanifolds in sequence, shadowing trajectories in the invariant submanifold[11].

In this talk we discuss the main scenario introduced by BCLN from a canonical perspective. This approach allows us to analyse in depth the physical properties of the classical solutions and to obtain a consistent quantum description of the model. We consider the bosonic sector of eleven-dimensional supergravity which consists of a

graviton and an antisymmetric three-form potential. The theory is compactified to four dimensions by assuming a geometry of the form  $M^4 \times T^6 \times S^1$ , where  $T^6$  is a six-dimensional torus and  $M^4$  corresponds to a spatially flat FRW spacetime. The effective theory in four dimensions bears a dilaton  $\phi$ , a modulus field  $\beta$  identifying the internal space, a pseudo-scalar axion field  $\sigma$  and a potential term induced by the RR four-form field. A brief derivation of the previous steps is presented in Section 2. In Section 3 we analyse the NSNS model, where the four-form field is negligible and the axion field dominates. In particular, in Subsection 3.1 we discuss the parameter space of the classical theory and in Subsection 3.2 we find the Hilbert space of states of the quantum theory. This programme is performed using a set of canonical variables, the so-called “hybrid” variables, that diagonalise the Hamiltonian. More details will appear in a forthcoming report[12]. Finally, our conclusions are presented in Section 4.

## 2 M-theory cosmology

In this section we derive[11] the four-dimensional minisuperspace effective action that will be used to discuss the dynamics of M-theory cosmology.

The bosonic sector of eleven-dimensional supergravity action  $S^{(11)}$  is

$$S^{(11)} = \int d^{11}X \sqrt{-g^{(11)}} \left[ R^{(11)}(g_{ab}^{(11)}) - \frac{1}{48} F_{a_1 \dots a_4} F^{a_1 \dots a_4} - \frac{1}{12^4 \sqrt{-g^{(11)}}} \epsilon^{a_1 \dots a_3 b_1 \dots b_4 c_1 \dots c_4} A_{a_1 \dots a_3} F_{b_1 \dots b_4} F_{c_1 \dots c_4} \right], \quad (1)$$

where  $a_i, b_i, c_i = 0 \dots 10$ ,  $F_{a_1 \dots a_4} = 4\partial_{[a_1} A_{a_2 \dots a_4]}$  is the four-form field strength of the antisymmetric three-form potential  $A_{a_1 \dots a_3}$ , and  $g^{(11)}$  denotes the determinant of the eleven-dimensional metric  $g_{ab}^{(11)}$ . Equation (1) describes the low-energy limit of M-theory.

The four-dimensional effective action is derived from Eq. (1) by a sequence of a Kaluza-Klein compactification on a circle  $S^1$  with radius  $R_{S^1} = e^{\Phi_{10}/3}$ , a conformal transformation of the ensuing ten-dimensional metric with conformal factor  $R_{S^1}^{-1}$ , and a further compactification on an isotropic six-torus with radius  $R_{T^6} = e^\beta$ .

We are interested in homogeneous and isotropic four-dimensional cosmologies. The ansatz for the four-dimensional section of the metric in the string frame is

$$ds_{(4)}^2 \equiv g_{\mu\nu} dx^\mu dx^\nu = -N^2(t) dt^2 + e^{2\alpha(t)} d\Omega_{3k}, \quad N(t) > 0 \quad (2)$$

where  $d\Omega_{3k}$  is a maximally symmetric three-dimensional metric with unit volume and curvature  $k = 0, \pm 1$ , respectively. Using Eq. (2) and requiring that the modulus field  $\beta$ , the dilaton  $\Phi_4$ , and the axion  $\sigma$  depend only on  $t$ , the four-dimensional effective action is

$$S = \int dt \left[ \frac{1}{\mu} \left( 3\dot{\alpha}^2 - \dot{\phi}^2 + 6\dot{\beta}^2 + \frac{\dot{\sigma}^2}{2} e^{2(3\alpha+\phi)} \right) + \mu \left( 6ke^{-2(\alpha+\phi)} - \frac{Q^2}{2} e^{3\alpha-\phi-6\beta} \right) \right], \quad (3)$$

where we have defined the “shifted dilaton” field

$$\phi = \Phi_4 - 3\alpha, \quad (4)$$

and the Lagrange multiplier

$$\mu(t) = Ne^\phi > 0. \quad (5)$$

The dynamics of the action (3) has been discussed qualitatively in Ref. [11]. Here we discuss in detail the model with  $Q = 0$  and  $k = 0$ . This case turns out to be completely integrable and describes spatially flat NSNS low-energy M-theory cosmology with negligible RR fields. The general solution for this model (including spatially curved models which are not discussed here) was first discussed by Copeland, Lahiri and Wands[9]. Let us note that the model with constant  $\sigma$  and  $k = 0$  is also completely integrable and has been discussed quantitatively in Ref. [12]. The latter case describes spatially flat low-energy M-theory cosmology with trivial axion and nonzero RR four-form.

### 3 NSNS low-energy M-theory cosmology

The action can be cast in the canonical form

$$S_I = \int dt \left[ \dot{\alpha} p_\alpha + \dot{\phi} p_\phi + \dot{\beta} p_\beta + \dot{\sigma} p_\sigma - \mathcal{H} \right], \quad (6)$$

where the Hamiltonian is

$$\mathcal{H} = \mu H, \quad H = \frac{1}{24} \left( 2p_\alpha^2 - 6p_\phi^2 + p_\beta^2 + 12p_\sigma^2 e^{-2(3\alpha+\phi)} \right). \quad (7)$$

The non-dynamical variable  $\mu$  enforces the Hamiltonian constraint

$$0 = 24H = 2p_\alpha^2 - 6p_\phi^2 + p_\beta^2 + 12p_\sigma^2 e^{-2(3\alpha+\phi)}. \quad (8)$$

The canonical equations of motion are

$$\begin{aligned} \dot{\alpha} &= \frac{p_\alpha}{6}, & \dot{\phi} &= -\frac{p_\phi}{2}, & \dot{\beta} &= \frac{p_\beta}{12}, & \dot{\sigma} &= p_\sigma e^{-2(3\alpha+\phi)}, \\ \dot{p}_\alpha &= 3p_\sigma^2 e^{-2(3\alpha+\phi)}, & \dot{p}_\phi &= p_\sigma^2 e^{-2(3\alpha+\phi)}, & \dot{p}_\beta &= 0, & \dot{p}_\sigma &= 0, \end{aligned} \quad (9)$$

where the dots represent differentiation w.r.t. gauge parameter

$$\tau(t) = \int_{t_0}^t \mu(t') dt', \quad t > t_0, \quad (10)$$

and  $t_0$  is an arbitrary constant. Note that since  $\mu$  is positive defined  $\tau(t)$  is a monotonic increasing function.

### 3.1 Classical solutions

Assuming  $p_\sigma \neq 0$  the off-shell solution of the canonical equations is

$$\begin{aligned}
\alpha &= \alpha_0 + \frac{1}{2} \ln [\cosh (\kappa(\tau - \tau_0))] - \xi(\tau - \tau_0), \\
p_\alpha &= 3\kappa \tanh [\kappa(\tau - \tau_0)] - 6\xi, \\
\phi &= \phi_0 - \frac{1}{2} \ln [\cosh (\kappa(\tau - \tau_0))] + 3\xi(\tau - \tau_0), \\
p_\phi &= \kappa \tanh [\kappa(\tau - \tau_0)] - 6\xi, \\
\beta &= \beta_0 + \frac{p_\beta}{12}(\tau - \tau_0), \\
p_\beta &= \text{constant}, \\
\sigma &= \sigma_0 + \frac{\kappa}{p_\sigma} \tanh [\kappa(\tau - \tau_0)] \\
p_\sigma &= \text{constant},
\end{aligned} \tag{11}$$

where  $\alpha_0, \phi_0, \beta_0, \sigma_0$  and  $\tau_0$  are constants of integration,

$$\kappa^2 - 12\xi^2 + \frac{p_\beta^2}{12} = 2H, \quad \kappa \neq 0, \tag{12}$$

and (we choose  $\kappa > 0$  for simplicity)

$$3\alpha_0 + \phi_0 = \ln \left( \frac{|p_\sigma|}{\kappa} \right). \tag{13}$$

A useful canonical chart is formed by the hybrid variables that diagonalise the constraint (8). Although the hybrid variables are not (all) gauge invariant they allow to fix a global gauge and quantize exactly the system. The hybrid variables  $(a, b, c, \sigma)$  are defined by the canonical transformation

$$\begin{aligned}
a &= \phi + 3\alpha, & b &= \sqrt{3}(\phi + \alpha), & c &= 2\sqrt{3}\beta, \\
p_a &= \frac{1}{2}(p_\alpha - p_\phi), & p_b &= \frac{1}{2\sqrt{3}}(3p_\phi - p_\alpha), & p_c &= \frac{1}{2\sqrt{3}}p_\beta.
\end{aligned} \tag{14}$$

Note that  $a$  coincides with the four-dimensional dilaton field  $\Phi_4$ . Using the hybrid variables the constraint (8) reads (we have divided by a factor 12)

$$p_a^2 - p_b^2 + p_c^2 + p_\sigma^2 e^{-2a} = 0. \tag{15}$$

Let us discuss the behaviour of the classical solution (11). The on-shell classical solution is determined by six physical parameters. Five of them ( $\alpha_0, \phi_0, \beta_0, \sigma_0$ , and  $\tau_0$ ) give

initial conditions for the canonical variables and will be set equal to zero. Therefore, the qualitative behaviour of the model is determined by a two-dimensional parameter space described by two coordinates, for instance  $\kappa$  and  $\xi$ . Using  $\kappa$  and  $\xi$  as free parameters, from Eq. (12) it follows that  $p_\beta$  is (on-shell)

$$p_\beta = \pm 2\sqrt{3}\sqrt{12\xi^2 - \kappa^2}. \quad (16)$$

The sign of  $p_\beta$  determines the dynamical behaviour of the internal six-torus space. From the solution of the equations of motion one obtains the scale factor of the internal space

$$R_{T^6} = e^{p_\beta \tau / 12}. \quad (17)$$

A successful physical model ultimately requires that the moduli fields are stabilized and compactified at late times. Stabilization of the internal space does not occur in the models under consideration, where only a fraction of all the degrees of freedom present in Eq. (1) are considered, with exception of the (fine-tuned) case  $p_\beta = 0$ . (Hopefully, the inclusion of more degrees of freedom will provide a mechanism for stabilization of extra-dimensions at late times.) Compactification of the six-torus space is achieved for  $p_\beta > 0$ . Indeed, for negative values of  $p_\beta$  the internal space shrinks to zero for large values of the gauge time  $\tau$  and decompactifies for  $\tau \rightarrow -\infty$  when the strong coupling region of the theory is approached. Since the relation between the comoving ( $N = 1$ ) time  $t$  and the gauge time is monotonic, the dynamics in  $\tau$  traces the dynamics in  $t$ , and the internal space shrinks to zero for large values of the (physical) comoving time as well. The fine-tuned limiting value  $p_\beta = 0$  corresponds to a constant (stable) internal space with unit radius. In the following we will restrict attention to nonpositive values of  $p_\beta$ .

At fixed  $\kappa$  we distinguish three different dynamical behaviours of the four-dimensional external space according to the value of  $\xi$ :

- i)  $\xi \leq -\kappa/2$  ( $p_\beta < -2\kappa$ ). In this case the external scale factor always expands while the internal scale factor shrinks from infinity to zero. The Hubble parameter is always positive and vanishes asymptotically at large times. In particular, for  $\xi = -\kappa/2$  the external space starts at  $\tau = -\infty$  with a finite nonzero scale factor and vanishing Hubble parameter. For  $\xi < -\kappa/2$  the external space starts with a vanishing scale factor and infinite Hubble parameter, which is always decreasing.  $\tau = -\infty$  is the strong coupling region where both the coupling constants of the theory,  $g = \exp(\phi)$  and  $g_{10} = \exp(\Phi_{10})$ , become infinite. Conversely,  $\tau = \infty$  is the weak region coupling where  $g$  and  $g_{10}$  vanish.  $g$  and  $g_{10}$  are always decreasing.
- ii)  $\xi \geq \kappa/2$  ( $p_\beta < -2\kappa$ ). In this case both the external scale factor and the internal scale factor always shrink. The Hubble parameter is always negative and asymptotically vanishing at small times. In particular, for  $\xi = \kappa/2$  the external space ends at  $\tau = \infty$  with a finite nonzero scale factor and vanishing Hubble parameter. For  $\xi > \kappa/2$  the external space ends with a vanishing scale factor and infinite Hubble parameter.  $g$  ( $g_{10}$ ) increases (decreases) from zero (infinity) to infinity (zero).
- iii)  $-\kappa/2 < \xi < -\kappa/2\sqrt{3}$  and  $\kappa/2\sqrt{3} < \xi < \kappa/2$ . In this case the external scale factor first contracts then expands, bouncing from infinity to infinity. In particular, for

- a)  $-\kappa/2 < \xi \leq -\kappa/3$  ( $p_\beta < -2\kappa$ ) the internal scale factor shrinks from infinity to zero. The Hubble parameter starts with infinite negative value, becomes positive and then decreases to zero after having reached a positive maximum.  $g$  and  $g_{10}$  decrease from infinity to zero [ $g_{10}$  to a finite nonzero positive value for the limiting value  $\xi = -\kappa/3$ ];
- b)  $-\kappa/3 < \xi \leq -\kappa/2\sqrt{3}$  the internal scale factor shrinks from infinity to zero [ $-\kappa/3 < \xi < -\kappa/2\sqrt{3}$  ( $-2\kappa < p_\beta < 0$ )] or is constant [ $\xi = -\kappa/2\sqrt{3}$  ( $p_\beta = 0$ )]. The Hubble parameter starts with infinite negative value, becomes positive and then decreases to zero after having reached a positive maximum.  $g$  decreases from infinity to zero.  $g_{10}$  bounces from infinity to infinity via a positive minimum;
- c)  $\kappa/2\sqrt{3} \leq \xi < \kappa/3$  ( $-2\kappa < p_\beta < 0$ ). The internal scale factor shrinks from infinity to zero [ $\kappa/2\sqrt{3} < \xi < \kappa/3$  ( $-2\kappa < p_\beta < 0$ )] or is constant [ $\xi = \kappa/2\sqrt{3}$  ( $p_\beta = 0$ )]. The Hubble parameter is first negative and small, decreases to a negative minimum and then increases to infinity.  $g$  increases from zero to infinity.  $g_{10}$  bounces from infinity to infinity via a positive minimum.
- d)  $\kappa/3 \leq \xi < \kappa/2$  ( $p_\beta < -2\kappa$ ). The internal scale factor shrinks from infinity to zero. The Hubble parameter is first negative and small, decreases to a negative minimum and then increases to infinity.  $g$  increases from zero to infinity.  $g_{10}$  decreases from infinity to zero for  $\kappa/3 < \xi < \kappa/2$  or to a finite nonzero positive value for the limiting value  $\xi = \kappa/3$ .

Scenarios *i*) and *iii*) may be suitable candidates for a physical description of a late time expanding universe emerging from a strong coupling region. According to *i*) a decelerated universe begins in a strong coupling region with large coupling constants,  $g$  and  $g_{10}$ , and internal dimensions much larger than the external dimensions. Though this might be seen as a kind of severe fine-tuned initial conditions, for early times we are in the strong coupling regime of the theory, where the spacetime curvature blows up, and we expect the low-energy description of M-theory to break down. Possibly, nonperturbative effects will cure initial conditions and provide a mechanism for early inflation. Inflation happens in case *iii*), where the external spacetime is first contracting and eventually expanding, thus evolving through an accelerated expanding phase. However, bouncing universes do not have sufficient inflationary e-foldings to solve the horizon problem. Indeed, we find

$$\frac{a_f}{a_i} = \left[ \frac{1}{2} \left( \sqrt{p^{-2} - 3} - 1 \right) \right]^{1/4} \left[ \frac{2p^2 - 1 + p\sqrt{p^{-2} - 3}}{p(2p + 1)} \right]^{-p/2}, \quad (18)$$

where  $p = \xi/\kappa$ , and  $a_i$  and  $a_f$  are the external scale factors at the beginning and at the end of the inflationary phase, respectively. For  $-1/2 < p < -1/(2\sqrt{3})$  and  $1/(2\sqrt{3}) < p < 1/2$  the ratio  $a_f/a_i$  is always finite and  $\approx \sqrt{2}$ .

### 3.2 Quantization

Turning to the hybrid canonical chart, from the constraint (15) it is natural to choose the operators  $\hat{p}_a$ ,  $\hat{p}_b$ ,  $\hat{p}_c$  and  $\hat{p}_\sigma$  as

$$\hat{p}_a = -i \frac{\partial}{\partial a}, \quad \hat{p}_b = -i \frac{\partial}{\partial b}, \quad \hat{p}_c = -i \frac{\partial}{\partial c}, \quad \hat{p}_\sigma = -i \frac{\partial}{\partial \sigma}. \quad (19)$$

The Wheeler-De Witt (WDW) equation is

$$\left[ -\frac{\partial^2}{\partial a^2} + \frac{\partial^2}{\partial b^2} - \frac{\partial^2}{\partial c^2} - e^{-2a} \frac{\partial^2}{\partial \sigma^2} \right] \Psi(a, b, c, \sigma) = 0. \quad (20)$$

The WDW equation can be completely solved by the technique of separation of variables. The general (bounded) solution is the superposition of wave functions

$$\begin{aligned} \Psi(a, b, c, \sigma) &= \int dk_b dk_c dk_\sigma A(k_b, k_c, k_\sigma) \psi(k_b, k_c, k_\sigma; a, b, c, \sigma), \\ \psi(k_b, k_c, k_\sigma; a, b, c, \sigma) &= \mathcal{N} e^{\pm i b k_b} e^{\pm i c k_c} e^{\pm i \sigma k_\sigma} K_{i\nu}(k_\sigma e^{-a}), \quad \nu = \sqrt{k_b^2 - k_c^2} \end{aligned} \quad (21)$$

where  $K_{i\nu}$  is the modified Bessel function of imaginary index  $i\nu$ . By properly choosing the normalization factor  $\mathcal{N}$ , and fixing the gauge using the  $b$  degree of freedom, the eigenstates of the physical Hamiltonian with energy  $E = k_b^2$  read

$$\psi_{k_b, k_c, k_\sigma} = \sqrt{\frac{\nu \sinh \pi \nu}{2\pi^4}} e^{\pm i c k_c} e^{\pm i \sigma k_\sigma} K_{i\nu}(k_\sigma e^{-a}). \quad (22)$$

Let us briefly discuss the correspondence between the hybrid wave functions and the classical solutions. The oscillating regions of the wave functions correspond to the classically allowed regions of the configuration space. Along  $c$  and  $\sigma$  directions the wave functions (22) are described by plane waves. Along the  $a$  direction the wave functions are oscillating in the region

$$0 < e^{-a} \lesssim \frac{\nu}{k_\sigma}. \quad (23)$$

This corresponds to the classically allowed region for the hybrid variable  $a$ . (We have chosen  $k_\sigma > 0$  for simplicity.) Indeed, from the solutions of the equations of motion we have

$$0 < e^{-a} = \frac{\kappa}{p_\sigma} [\cosh(\kappa \tau)]^{-1} \leq \frac{\kappa}{p_\sigma}. \quad (24)$$

The wave functions go like  $e^{\pm i a \nu}$  for large values of  $a$ . Finally, the relation between the quantum numbers  $k_i$  and the classical parameters that characterize the behaviour of the classical solution is

$$k_b = -2\sqrt{3}\xi, \quad k_c = \frac{p_\beta}{2\sqrt{3}}, \quad \nu = \kappa. \quad (25)$$



## 4 Conclusions

In this talk we have analysed a simple spatially flat, four-dimensional cosmological model derived from the M-theory effective action, Eq. (1). The eleven-dimensional metric is first compactified on a one-dimensional circle to obtain the type IIA superstring effective action and then on a six-torus to obtain the effective four-dimensional theory. In our investigation we concentrated the attention on the boundary of the physical phase space of the theory, and in particular to the invariant submanifold with negligible RR four-form field strength. In our discussion we have heavily employed the canonical formalism. This approach makes the analysis of the features of the classical solution extremely simple and allows a straightforward quantization of the theory.

In the classical setting, we have found regions in the moduli space where a four-dimensional FRW universe evolves from a strong coupling regime towards a weak coupling regime, both internal six-volume and eleven-dimension contracting. The dynamics may also be characterized by an early accelerated (inflationary) expansion with the spacetime eventually approaching a standard FRW decelerated expansion.

The quantization of the two invariant submanifolds can be performed exactly and the Hilbert space of states can eventually be obtained. In the quantum framework our analysis allows to identify the quantum states that correspond to the different classical behaviours. In the hybrid representation we have identified regions in the space of parameters where the wave function of the universe is either oscillating or exponentially decaying. These regions are determined by the inverse exponential function of the four-dimensional (unshifted) dilaton, i.e., by the four-dimensional string coupling, and correspond to classically allowed and classically forbidden regions, respectively. Starting from the Hilbert space of states, the quantum mechanics of M-theory cosmology can be constructed with aid of usual elementary quantum mechanics techniques.

## Acknowledgments

We are very grateful to M. Gasperini, M. Henneaux, N. Kaloper, J. Lidsey, C. Ungarelli, A. Vilenkin and D. Wands for interesting discussions and useful comments. This work is supported by grants ESO/PROJ/1258/98, CERN/P/FIS/15190/1999, Sapiens-Proj32327/99. M.C. is partially supported by the FCT grant Praxis XXI BPD/20166/99. This work is supported in part by funds provided by the U.S. Department of Energy (D.O.E.) under cooperative research agreement DE-FC02-94ER40818.

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